EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Additional Mathematics 4030/1
Paper 1

Monday 14 NOVEMBER 2016

Additional Material(s):
Answer Booklet; Graph paper (1 Sheet)
Electronic calculators

Time: 2 hours

Instructions to Candidates

Write your name, centre number and candidate number in the spaces on the Answer Booklet provided.

There are 12 questions in this paper. Answer all questions.

Write your answers in the Answer Booklet provided.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Information for candidates

The number of marks is shown in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The use of a non programmable electronic calculator is expected, where appropriate.

Cell phones are not allowed in the examination room.

You are reminded of the need for clear presentation in your answers.

Check the formulae overleaf.
MATHEMATICS FORMULAE

1 ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2 TRIGONOMETRY

Identities

$\sin^2 A + \cos^2 A = 1$

$\sec^2 A = 1 + \tan^2 A$

$\csc^2 A = 1 + \cot^2 A$

Formulæ for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$
1 The line $2y + 3x = 12$ cuts the x-axis and y-axis at A and B respectively. Calculate the coordinates of the mid-point of A and B. [4]

2 Solve the simultaneous equations
   
   \[2y - x = 4,\]
   \[xy = 4 + 3x.\] [5]

3 Find the range of values of $k$ for which the line $y = -3x + 2k + 2$ does not meet the curve $y = x^2 + 3x - 5$. [4]

4 A function $f$ is defined by $f: x \rightarrow \frac{3x - 1}{2}$.

   Find
   (a) the value of $a$ given that $f(f(a)) = \frac{5}{8}$, [3]
   (b) the value of $x$ such that $f^{-1}(x) = f(x)$. [3]

5 In the diagram below, OABC is a rhombus. O is the centre of a circle through A, B and C. $\angle AOC = \frac{2\pi}{3}$ and OA = 8cm. The diagonals OB and AC intersect at right angles at P.

   Find
   (a) the perimeter of the shaded region. [3]
   (b) the area of the shaded region. [2]
6 Prove the identity \( \frac{2\sin^2 \theta - 1}{\sin \theta \cos \theta} \equiv \tan \theta - \cot \theta \). [4]

7 (a) The first three terms in the expansion of \((1 + ax)^n\) are \(1 - 16x + 112x^2\). Find the values of \(a\) and \(n\). [4]

(b) Given that the coefficient of \(x^3\) in the expansion of \((3 + kx)(1 - x)^7\) is zero, find the value of \(k\). [5]

8 (a) Find an expression for \(\frac{dy}{dx}\) of the equation \(y = (2a^2 + bx)^4\), where \(a\) and \(b\) are real. [3]

(b) Two variables \(x\) and \(y\) are related by the equation \(y = \left(\frac{x}{2} - 4\right)^3\). Obtain an expression for \(\frac{dy}{dx}\) and find the approximate change in \(y\) as \(x\) increases from 4 to 4.02. [6]

9 Find all the angles between \(0^\circ\) and \(360^\circ\) which satisfy the equation \(|\tan x + 1| = 3\). [5]

10 (a) The straight lines \(a = (2i + j) + \lambda(i + j)\) and \(b = (5i + j) + \mu(2i - j)\) intersect at the point \(P\). Find the coordinates of \(P\). [4]

(b) The position vectors of the points \(A\) and \(B\) with respect to \(O\) are \(4i - 3j\) and \(-i + 2j\) respectively. Find the position vector of the point \(C\) such that \(\overrightarrow{AC} = 3\overrightarrow{AB}\). [5]
11 (a) Evaluate \( \int_0^\pi \sin x \, dx \). \[2\]
(b) The diagram below shows part of the curve \( y = 2x - x^2 \) intersecting the curve \( y = x^2 - x \) at O and A.

\[\text{Find} \]

(i) the coordinates of A. \[2\]
(ii) the area of the shaded region. \[6\]

Answer only one of the following alternatives.

12 Either

Solutions to this question by accurate drawing will not be accepted.

In the diagram below, \( \text{ABCD} \) is a trapezium. \( \text{AB} \) is parallel to \( \text{DC} \) and \( \angle \text{DAB} = 90^\circ \). The coordinates of \( \text{A} \) and \( \text{D} \) are \((-4, 4)\) and \((2, 8)\) respectively. The equation of \( \text{BC} \) is \( 4y = 7x - 8 \).

\[\text{Find} \]

(a) the equation of \( \text{AB} \), \[4\]
(b) the equation of \( \text{DC} \), \[3\]
(c) the coordinates of \( \text{C} \). \[3\]
Or

The diagram below shows a rectangle ABCD. M is the mid point of AB and AB = DC = 20cm. The points E and F are on AD and BC respectively, such that AE = BF = 7cm. EF is an arc of a circle centre M, such that angle EMF is \( \theta \) radians.

Find

(a) the value of \( \theta \), \([3]\)
(b) the perimeter of the shaded region, \([4]\)
(c) the area of the shaded region. \([3]\)